PMM U.S.S.R., Vol.45,pp.77-84
Copyright Pergamon Press Ltd.1982.Printed in U.K.

0021-8928/82/1 0077 $7.50/0

UDC 532.72

ON THE CONCENTRATION FIELD OF AN ORDERLY SYSTEM OF REACTING PLATES DISTRIBUTED ALONG A STREAM *

A. D. POLIANIN and Iu. A. SERGEEV

The plane problem of convective diffusion in a semi-infinite ordered system of plates over which flows lengthwise a stream of perfect liquid is considered. The Péclet number determined relative to the length of a plate is assumed high hence the variation of the concentration field is primarily dependent on the interaction of diffusion wakes and boundary layers of plates lying one after another along the stream /1—4/.

The object of this work is to derive and investigate the equations that define the concentration distribution at the outer boundary of each plate boundary layer. It is shown that in such systems the distribution of concentration in the stream core is defined by an integral equation of the Volterra type with an integrable singularity. An exact solution is obtained in the case of a single chain of plates, and the asymptotics of solution away from the system entry is determined for a periodic system of chains parallel to each other. A method of successive approximations which in certain cases makes possible the determination of asymptotics of the concentration field is proposed. Analysis of asymptotics of the exact solution for a single chain of plates shows that the method of successive approximations yields an exact value for the principal term of the asymptotic expansion.

The proposed methods is extended to nonlinear problems and makes possible the formulation of respective equations and boundary conditions. The flow of a viscous fluid at high Reynolds numbers past a periodic system of plates is investigated as an example.

The process of convective diffusion was previously considered in /1/ and /2/ in thinly scattered and in concentrated lattices of reacting spheres, and in /3,4/ the diffusion to chains of solid reacting particles was investigated.

1. Statement of the problem. Let us consider a system of plates of length l periodically distributed in a uniform stream of perfect incompressible fluid flowing in the lengthwise direction. The system is of periods a and b, respectively, along the y- and x-axes (Fig.1). We assume that a reaction with total absorption of the substance dissolved in the fluid takes place at the plate surfaces. Concentration of the substance at some distance upstream of the system intake is constant.

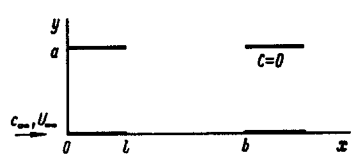

Fig.1

The described model can be used for defining processes of diffusion or heat conduction, when a system of plates (for instance, inserts in a chemical reactor) in a stationary or fluidized layer of particles through which filters a stream of fluid. In such case the flow field may be considered potential. The convective heat transfer to the walls of a reactor with a stationary or fluidized layer of particles, and with heating or cooling elements periodically distributed in the walls, is another example. In this formulation of the problem it is possible to simulate also heat and mass transfer systems in the presence of areas of corrosion on working surfaces /5/.

At high Péclet numbers at which only transverse transfer of substance in diffusion boundary layers and late wakes is significant, the distribution of concentration in the stream is determined by the solution of the following boundary value problem:

$$\mathbf{L}c = 0,\ \mathbf{L} = \partial/\partial x - \partial^2/\partial y^2 \tag{1.1}$$

$$x = 0,\ c = 1 \tag{1.2}$$

$$c(y + a) = c(y) \tag{1.3}$$

$$\begin{gathered} y = ma,\ x = kb + l\tau,\ c = 0 \\ y = ma,\ x = kb + l + (b - l)\tau,\ \partial c/\partial x = 0 \\ k = 0, 1, 2, \ldots;\ m = 0, \pm 1, \pm 2, \ldots;\ \tau \in [0, 1] \end{gathered} \tag{1.4}$$

*Prikl.Matem.Mekhan.,45,No.1,105-115,1981 where the x-coordinate is measured from the leading edge of the first plate, k is the plate ordinal number; the fluid stream velocity and the coefficient of diffusion are assumed equal unity (which can be always achieved by a proper choice of the scale of the variable y). Equation (1.1) determines the principal term of the respective asymptotic expansion for the concentration (with respect to the high Péclet number) and is valid throughout the flow region, since for plates oriented along the stream the equations of the diffusion wake and of the boundary layer are the same (the proof is similar to that in /6/).

In the case of a single chain of plates the periodicity condition (1.3) is replaced by the concentration uniformity condition

$$y \to \infty, \quad c \to 1 \tag{1.5}$$

and the boundary condition (1.4) is specified for only one $m = 0\,(y = 0)$.

Below, we assume the lengthwise period of the lattice to satisfy the condition $T = lb^{-1} \ll 1$.

For a fairly extended system of plates $(k \gg 1)$ diffusion in boundary layers of individual plates may result in a substantial change of concentration outside boundary layers from the concentration $c = 1$ at the system intake. At transition from one plate to another the concentration changes on the outer boundary of the layer by the quantity $\sqrt{T} \ll 1$. But even small concentration changes on sections of the order of the lattice period b result in a substantial concentration change at some distance from the system intake $(k \gg 1)$.

The basic aim of the present work is the derivation and investigation of equations that define concentration distribution at the outer boundary of each plate boundary layer, when numerous plates $(k \gg 1)$ lie within the characteristic length scale.

2. Diffusion on a single chain and on a periodic system of plates. As in the analysis of viscous incompressible fluid flow over a plate at high Reynolds numbers /6/, it can be shown that the maximum thickness of the first plate boundary layer is $\delta_1 = lO\,(P^{-1/2})$ and that of its diffusion wake around the second plate of the chain is $\delta_1{}^* = T^{-1/2}\delta_1 \gg \delta_1$. It follows from this and the concentration distribution in the diffusion wake of a single plate that when $T \ll 1$, the concentration $c\,(x, y)$ in the neighborhood of the second plate, but outside its diffusion boundary layer, varies slowly; any of the plates has this property.

The equation for concentration distribution outside diffusion layers of plates can be obtained by adding the point sources which define concentration distribution in diffusion wakes of plates (when $T \ll 1$) and located at their centers, as was done in /1/ in the case of a sphere lattice. Here we propose a more general approach based on the use of the input equation (1.1) and the derivation of a supplementary integral boundary condition ("condition of conservation" of the reacting substance in a volume) for concentration. Although the final formulas for concentration distribution obtained by the proposed here method can be obtained using the procedure in /1/, it naturally extends to nonlinear problems (e.g., flow of viscous fluid at high Reynolds numbers over plates, considered in Sect.3) and makes possible readily to formulate respective equations and boundary conditions (for velocity components) in the general case.

To simplify reasoning we conditionally separate below the outer region Ω with large gradients of concentration c. Region Ω is obtained by excluding in the half-plane $x \geqslant 0$ of the diffusion boundary layer the parts of diffusion wakes (of length $O\,(1)$) lying immediately behind plates, where an abrupt concentration change takes place (Fig.2). In fact Ω coincides with the region of validity of representing concentration distribution in a system of plates by point sources /1/. We denote the concentration in region Ω by $C\,(x, y)$. For $T \ll 1$ the external concentration does not change much over the lattice period $C\,(x + b, y) - C\,(x, y) = O\,(\sqrt{T})$, and the basic variation of concentrations is due to the absorption of substance on the reacting plates induced by the diffusion wakes behind them. These wakes decrease concentration at the external boundary of the next following plate boundary layer.

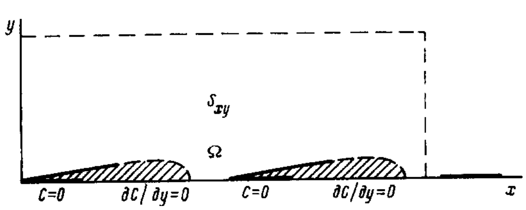

Fig.2

Since in the outer region concentations c and C are equal, the external concentration $C\,(x, y)$ satisfies in region Ω Eq.(1.1) and boundary conditions (1.2) and (1.5). To complete the formulation of the problem of concentration C it is necessary to add one more boundary condition of the "law of conservation" type, which contains information on boundary conditions (1.4) which are now not satisfied for C. The necessary (additional) boundary conditions can be obtained using a reasoning of the type implicitly used in /1/ for the derivation of the integral equation for concentration when determining unknown coefficients that appear as multipliers in self-similar solutions that correspond to point sources at sphere centers.

To obtain the last boundary condition we integrate, as in /1/, Eq.(1.1) over the reference volume $S_{xy} = \{[0,\ x];\ y \in [0,\ +\infty)\}$ (Fig.2)

$$\int_0^x \int_0^\infty \mathrm{L}c\, dy\, dx = -\int_0^\infty [1 - c(x, y)]\, dy + \int_0^x \left[\frac{\partial c}{\partial y}\right]_{y=0} dx = 0 \tag{2.1}$$

The integral identity (2.1) represents the law of conservation of mass of the diffusing substance in the volume S_{xy}. If there are k plates at distance x from the system intake, i.e. $(k-1)\, b < x < kb$, then by virtue of the boundary conditions (1.4) we have the equality

$$\int_0^x \left[\frac{\partial c}{\partial y}\right]_{y=0} dx = \sum_{n=0}^{k-1} I_{n+1}, \quad I_{n+1} = \int_{nb}^{nb+l} \left[\frac{\partial c}{\partial y}\right]_{y=0} dx \tag{2.2}$$

where I_n is the total diffusion flux to the n-th plate.

Since concentration $C(x, y)$ varies only little over distance b in the plate neighborhood, the concentration distribution in the boundary layer of the n-th plate can be represented as /6/

$$c_n(x, y) = C_n(x, 0)\, \mathrm{erf}\left(\frac{y}{2\sqrt{x_n'}}\right) \tag{2.3}$$

where $x_n' = x - nb$ is the coordinate measured from the n-th plate leading edge.

Calculating integral (2.2) with allowance for (2.3), we obtain

$$\int_0^x \left[\frac{\partial c}{\partial y}\right]_{y=0} dx = \sum_{n=0}^{k-1} C_n(x, 0) \cdot 2\sqrt{\frac{l}{\pi}} \approx 2\sqrt{\frac{l}{\pi}}\, \frac{1}{b} \int_0^x C(\tau, 0)\, d\tau \tag{2.4}$$

where the integral is substituted for the sum, since concentration changes little over a distance of the order of the lattice period b, i.e. $C_n - C_{n-1} = o(1)$. Taking into account the thinness of the diffusion boundary layer and formula (1.6), from (2.1) we obtain

$$\int_0^\infty [1 - C(x, y)]\, dy - \lambda \int_0^x C(\tau, 0)\, d\tau = 0, \quad \lambda = 2\pi^{-1/2} l^{1/2} b^{-1} \tag{2.5}$$

Consequently concentration $C(x, y)$ satisfies in the external region Ω the boundary value problem

$$\mathrm{L}C = 0;\ x = 0,\ C = 1;\ y \to \infty,\ C \to 1 \tag{2.6}$$

$$-\frac{\partial}{\partial x} \int_0^\infty C(x, y)\, dy = \lambda C(x, 0) \tag{2.7}$$

of which the last is obtained by differentiating Eq.(2.5). The right-hand side of that condition represents the diffusion flux to plates per unit length of the system.

We seek a solution of problem (2.6), (2.7) of the form

$$C(x, y) = 1 - \int_0^x \frac{A(x')}{\sqrt{x - x'}} \exp\left[-\frac{y^2}{4(x - x')}\right] dx' \tag{2.8}$$

Function (2.8) satisfies the equation and two boundary conditions (2.6) for any smooth core $A(x)$ /7/. Substituting expression (2.8) into the last of boundary conditions (2.7) and integrating with respect to y we obtain the relation

$$\lambda C(x,\ 0) = \sqrt{\pi} A(x) \tag{2.9}$$

Substitution of this equality into (2.8) and assuming in it $y = 0$ yields the equation for concentration $C(x, 0) \equiv C(x)$ on the inner boundary of the external region Ω

$$C(x) = 1 - \frac{\lambda}{\sqrt{\pi}} \int_0^x C(x') \frac{dx'}{\sqrt{x - x'}} \tag{2.10}$$

It should be noted that Eq.(2.10) can also be obtained by adding point sources located at plate centers, which specify concentration distribution in the distant region of diffusion wakes behind these, with subsequent substitution of an integral for such sum, as was done in

/1/. Equation (2.10) has the following exact solution /8/:

$$C(x)=1+\int_0^x g(x',\lambda)\,dx', \quad \lambda=2\pi^{-1/2}l^{1/2}b^{-1}, \quad g(x,\lambda)=-\frac{\lambda}{\sqrt{\pi x}}+\frac{\lambda^2}{\sqrt{\pi}}\exp(\lambda^2 x)\,\Gamma\left(\frac{1}{2},\lambda^2 x\right) \tag{2.11}$$

where $\Gamma(1/2, x)$ is an incomplete gamma function. It follows from formula (2.11) that the asymptotics of function $C(x)$ for large x is of the form

$$x\to\infty,\ C(x,0)\to {}^1/_2 b l^{-1/2} x^{-1/2} \tag{2.12}$$

which shows that away from the chain beginning, concentration on the outer boundary of the diffusion boundary layer of the plate approaches zero as $x^{-1/2}$ (inversely proportional to $\sqrt{k}$, $k\to\infty$).

Let us now consider convection diffusion in a system of recurrent plates. The problem was formulated in Sect.1 and is defined by Eq.(1.1) with boundary conditions (1.2)–(1.4).

We introduce region Ω (Fig.2) and shall consider the problem of determining the external field of concentration $C(x, y)$ (concentration outside the diffusion boundary layer). Here, as in the case of a single chain of plates, the external concentration satisfies Eq.(1.1) and boundary conditions (1.2) and (1.3); to obtain the last boundary condition we integrate Eq. (1.1) (taking into account all boundary conditions) along the contour $S_{xy}=\{[0, x], y\in[0, a/2]\}$.

In the integration allowance is made for the problem symmetry relative to the straight line $y=a/2$ by virtue of which the equality $[\partial c/\partial y]_{y=a/2}=0$ is satisfied.

Assuming that the interval $[0,x]$ contains k plates, we obtain the law of conservation of the mass of diffusing substance in the form (2.1), (2.2), but with $a/2$ substituted for ∞ as the upper limit of integration.

Taking into account that concentration varies only little in region Ω over the distance b between adjacent plates, we conclude that the concentration distribution in the boundary layer of the n-th plate is defined by (2.3). Calculating now the last integral in (2.1) for $k\gg 1$, we obtain the approximate equality (2.4). Taking into account (2.1) and (2.4), we obtain for concentration $C(x, y)$ in the external region Ω the equality (2.5) in which the upper limit of integration with respect to the transverse coordinate is $a/2$.

For the concentration $C(x, y)$ in the external region in the first quadrant $x\geqslant 0$, $y\geqslant 0$ ($C(x, y)=C(x, -y)$) we finally obtain the boundary value problem

$$\mathbf{L}C=0;\ x=0,\ C=1;\ C(y+a)=C(y) \tag{2.13}$$

$$-\frac{\partial}{\partial x}\int_0^{a/2} C(x,y)\,dy=\lambda C(x,0) \tag{2.14}$$

where the coefficient λ is determined in formula (2.5).

We seek a solution of problem (2.13) of the form

$$C(x,y)=1-\int_0^x \frac{A(x')}{\sqrt{x-x'}}\left(\sum_{m=-\infty}^{+\infty}\exp\left\{-\frac{(y-ma)^2}{4(x-x')}\right\}\right)dx' \tag{2.15}$$

Function (2.15) is periodic with respect to the y-coordinate of period a, and for any core $A(x)$ satisfies the equation and the first of boundary conditions (2.14).

The substitution of expression (2.15) into the boundary condition (2.14) followed by integration with respect to y yields the link between the core $A(x)$ and concentation along the axis $C(x, 0)$, leading to formula (2.9).

Taking this into account and making y approach zero in (2.15), we obtain for concentration on the external boundary of diffusion boundary layers of plates (at the inner boundary of region Ω) the integral equation

$$C(x)=1-\frac{\lambda}{\sqrt{\pi}}\int_0^x \frac{C(x')}{\sqrt{x-x'}}\,\Theta_3\left(\frac{a^2}{4\pi(x-x')}\right)dx' \tag{2.16}$$

$$\Theta_3(x)=\sum_{m=-\infty}^{+\infty}\exp(-\pi m^2 x), \quad C(x)\equiv C(x,0)$$

where $\Theta_3(x)$ is the theta function /9/.

As $a\to\infty$ Eq.(2.16) reduces to Eq.(2.10) for a single chain of plates.

Let us investigate Eq.(2.16). For the Laplace transform $C(p)$ of concentration $C(x)=C(x, 0)$ we have in conformity with the theorem on convolution we have the equation

$$C(p)=\frac{1}{p}-\lambda C(p)\sum_{m=-\infty}^{\infty}\frac{1}{\sqrt{p}}\exp(-a|m|\sqrt{p})$$

where, and in what follows the originals and transforms are distinguished by the arguments x and p, respectively. Carrying out summation for the concentration transform on the axis, we obtain

$$C(p)=\left[p+\lambda\sqrt{p}\,\mathrm{cth}\left(\frac{1}{2}a\sqrt{p}\right)\right]^{-1} \tag{2.17}$$

Formula (2.17) enables us to obtain asymptotics of distribution of concentration $C(x)$ on the axis for large x. When the distance a between plate chains is large, the inequality $a^{-2}\ll p\ll 1$ for the Laplace variable p corresponds to the intermediate asymptotics for $1\ll x\ll a^2$. Retaining in the denominator in (2.17) the principal terms of expansion in $p^{1/2}$, we obtain the intermediate asymptotics in the form (2.12) which is the same as the asymptotic expression for concentration along the axis of a single chain of plates. For the determination of intermediate asymptotics only the term $\lambda\sqrt{p}\,(\mathrm{cth}\,(^1/_2 a\sqrt{p})\approx 1)$ is in this case essential in the denominator of formula (2.17), which corresponds to the neglect of the left-hand side of the input equation (2.16).

Note that physically the intermediate asymptotics corresponds to a region fairly remote from the system intake, where the effect of adjacent plates of the chain does not manifest itself.

For $x\gg a^2$ $(p\ll a^{-2})$, taking into account that $\mathrm{cth}\,(a\sqrt{p}/2)=2a^{-2}p^{-1/2}(1+a^2p/12+O(p^2))$, we obtain from formula (2.17) the asymptotic expression

$$C(x)=q\exp[-2\lambda qa^{-1}x]\quad(x\to\infty),\qquad q=(1+{}^1/_6 a\lambda)^{-1} \tag{2.18}$$

where a is arbitrary.

Formula (2.18) implies that the effect of adjacent chains leads to a considerably more important decrease of concentration along the axis. Note that both terms of the denominator of (2.17) are essential in the derivation of the "distant" asymptotics (2.18), i.e. that both sides of Eq. (2.16) are of the same order of smallness.

The used here method is readily extended to nonlinear problems and makes possible the formulation of respective equations and boundary conditions. As an example, we shall consider the flow of a viscous fluid at high Reynolds numbers over a periodic system of plates.

3. Flow of viscous incompressible fluid at high Reynolds numbers past a periodic system of plates. Let us consider the flow of a uniform at infinity stream of viscous incompressible fluid past a system of plates described in Sect.1. The Reynolds number of the flow past a single plate is assumed high.

The flow field is determined by the solution of the boundary value problem for the equation of boundary layer

$$\mathbf{G}(\mathbf{u})=(\mathbf{u}\nabla)u_1-\nu\partial^2u_1/\partial y^2=0 \tag{3.1}$$

$$\mathrm{div}\,\mathbf{u}=0;\quad \mathbf{u}=\{u_1,\ u_2\}$$

$$x=0,\quad u_1=U,\quad u_2=0 \tag{3.2}$$

$$\mathbf{u}(x,\ y+a)=u(x,\ y) \tag{3.3}$$

$$y=ma,\ \begin{cases}x=kb+\tau l, & u_1=0,\quad u_2=0\\ x=kb+l+(b-l)\tau, & \partial u_1/\partial x=0,\quad u_2=0\end{cases} \tag{3.4}$$

(in this case the boundary layer and the wake equations coincide (Fig.1). In these equations u_1 and u_2 are the longitudinal and transverse velocity components, respectively, ν is the kinematic viscosity of the fluid, and U is the stream velocity at infinity.

To obtain the distribution of velocities v_1 and v_2 in the external region we use, as in Sect.2, the integral law of momentum conservation which is obtained by integrating Eq.(3.1) of motion over the reference volume S_{xy}, taking into account the equation of continuity. For this formula

$$u_{1k}=\alpha y v_{1k}^{3/2}(\nu x)^{-1/2},\quad \alpha=0{,}332\quad(y\to 0) \tag{3.5}$$

for the distribution of the longitudinal velocity component in the boundary layer of the k-th plate /6/ is to be used (v_{ik} are velocity components in the Ω région).

The procedure for obtaining equations for the external region Ω is the same as in Sect. 2, and yields the following boundary value problem:

$$G(\mathbf{v}) = 0, \ \operatorname{div} \mathbf{v} = 0; \ \mathbf{v} = \{v_1, v_2\} \tag{3.6}$$

$$x = 0, \ v_1 = U, \ v_2 = 0; \ \mathbf{v}(y + a) = \mathbf{v}(y)$$

$$-\frac{\partial}{\partial x} \int_0^{a/2} v_1^2(x, y)\, dy = \sigma v_1^{3/2}(x, 0) \tag{3.7}$$

$$y = 0, v_2 = 0; \quad \sigma = 2\alpha \nu^{-1/2} l^{1/2} b^{-1}$$

In the case of a single chain of plates the condition of periodicity with respect to the transverse coordinate is replaced by the homogeneity condition

$$y \to \infty, \ v_1 = U, \ v_2 = 0 \tag{3.8}$$

with parameter a appearing in the upper limit of integration in the boundary condition (3.7) for v_1 assumed equal infinity.

4. The method of successive approximations. It will be shown here that in certain cases the use of the method of successive approximations makes it possible to determine the asymptotics for the distribution of concentration $C(x, 0)$ (or the longitudinal velocity component) at the external diffusion (hydrodynamic) boundary layer of a plate, as $x \to \infty$ directly from the boundary value problems (2.6), (2.7) and (2.13), (2.14) (or (2.16)) without using Eqs.(2.10) and (2.16).

Let us consider the boundary value problem for the unknown function w in the region $x \geqslant 0, 0 \leqslant y \leqslant a$ (where a is arbitrary and in particular $a = \infty$)

$$\Lambda w = 0 \tag{4.1}$$

$$w(\Gamma_i) = w_i, \ i = 1, 2 \tag{4.2}$$

$$w(x, 0) = F(w) \tag{4.3}$$

where Λ is some (nonlinear) parabolic operator and (4.2) and (4.3) are boundary conditions for Eq.(4.1). For instance, $\Lambda = L$ can be defined in the form (1.1) with boundary conditions defined by (1.2) and (1.3) or by (1.3) and (1.5); (4.3) is a complex boundary condition of the form (2.7) or (2.14), it may even have a more complex structure, and can be nonlinear, as in (3.7).

Let us assume that

$$\lim_{x \to \infty} w(x, 0) = 0 \tag{4.4}$$

is known with respect to problem (4.1)—(4.3) (for instance from the physical formulation of the problem) and it is required to determine the first term of the asymptotic expansion of $w(x, 0)$ as $x \to \infty$. For this we consider the auxilliary problem for function w_*

$$\Lambda w_* = 0; \ w_*(\Gamma_i) = w_i; \ w_*(x, 0) = 0 \tag{4.5}$$

This problem differs from the input problem (4.1)—(4.3) by the substitution of the simplest asymptotic boundary condition (4.4) for the complex boundary condition (4.3).

The solution of problem (4.5) is usually constructed in a simpler manner than that of problem (4.1)—(4.3), and in many cases can be obtained in explicit analytic form. The principal term of asymptotic expansion for $w(x, 0)$ as $x \to \infty$

$$x \to \infty, \ w(x, 0) = F(w_*(x, y)) \tag{4.6}$$

The indicated procedure of derivation of the principal term of the asymptotics of problem (4.1)—(4.3) as $x \to \infty$, can in certain cases fully substantiated.

The formulation of problem (2.6), (2.7) of diffusion on a chain of reacting plates and the method of obtaining equations for concentration on the external boundary of diffusion boundary layers of plates (2.1) imply that the determination of asymptotics of concentration $C(x)$ using formulas (4.6) exactly corresponds to the asymptotics of solutions of curtailed integral equations (2.10) (in whose left-hand side $C(x) = 0$ is set). On the other hand, the curtailed equation as $x \to \infty$ yields correct asymptotics for the input equations in the case of many integral equations.

In the case of solution $w(x) = w(x, \omega)$ of the integral equation of the form

$$\omega w(x) = 1 - \lambda_* \int_0^x \frac{W^\nu(x')\, dx'}{(x - x')^\beta}, \qquad (\lambda_* > 0, \ \nu > 0; \ 0 < \beta < 1) \tag{4.7}$$

in conformity with the procedure in /10/, where the case of $\nu \geqslant 1, \beta = 1/2$ was considered, it is possible to show that the following limit equality applies:

$$\lim_{x \to \infty} w(x, 1)\, w^{-1}(x, 0) = 1$$

Setting in Eq. (4.7) $\omega = 1, \nu = 1, \lambda_* = \lambda\pi^{-1/2}, \beta = 1/2$, we obtain Eq. (2.10). A direct substitution of the asymptotics for $C(x, 0)$, as $x \to \infty$, defined by formula (2.12) shows that it is an exact solution of the curtailed equation (2.10) (and corresponds to the zero value of parameter ω in (4.7).

Introduction of the auxilliary function and derivation of asymptotics of the input problem, using formula (4.6) may be treated as a certain modification of the method of successive approximations at whose first stage a solution of w_* which is simpler than the input problem (4.6) followed by its substitution into the right-hand side of the complex input boundary condition (4.3) and separation in $F(w_*)$ the principal term of asymptotic expansion as $x \to \infty$.

Let us illustrate the use of formula (4.6) on problems (2.6), (2.7) and (3.6)—(3.8).

1°. In the first case solution of the auxilliary problem for concentration by virtue of the last simplified boundary condition (4.5)

$$c_*(x, 0) = 0 \tag{4.8}$$

shows that the auxilliary function $c_*(x, y)$ is the self-similar solution of the problem of diffusion on semi-infinite plate under conditions of total absorption at its surface. Hence for c_* we have

$$c_*(x, y) = \operatorname{erf}(y/2\sqrt{x}) \tag{4.9}$$

Substitution of this expression into formula (4.6) with the explicit form of functional F (2.7) taken into account yields the equality

$$C(x, 0) = -\frac{1}{\lambda}\int_0^\infty \frac{\partial}{\partial x} c_*(x, y)\, dy = \frac{1}{\lambda}\sqrt{\frac{1}{\pi x}} \quad (x \to \infty)$$

where the parameter λ has been defined in (2.5). This formula coincides exactly with asymptotics (2.12) obtained from the exact solution of problem (2.11).

Remark. The integral in formulas (2.7) and (4.6) can be calculated more simply without taking into account the explicit form of the auxilliary function (4.9) by using the equality

$$\left[\frac{\partial c_*}{\partial y}\right]_{y=0} = -\frac{\partial}{\partial x}\int_0^\infty c_*(x, y)\, dy$$

that follows from the law of conservation of substance for c_* obtained, for example, from the integral identity (2.1) by the substitution of c_* for c. Using the integral identity (of type (2.1)) we calculate the right-hand side of (4.6) in a similar manner also in the general case.

2°. In the second case solution $u_{1*}(x, 0) = 0$ for the flow field in a system of plates shows, by virtue of condition (4.5), that the auxillary function $u_{1*}(x, y)$ is a self-similar solution of the boundary value problem of flow of viscous incompressible fluid over a semi-infinite plate /6/. For u_{1*} we consequently have

$$y \to 0, \; u_{1*}(x, y) = \alpha y U^{3/2} (\nu x)^{-1/2} \tag{4.10}$$

It can be shown similarly to 1° that the integral in the right-hand side of boundary condition (3.7) is determined by the quantity $[\partial u_{*1}/\partial y]_{y=0}$ which with allowance for (4.10) results in the following asymptotic law of damping of the longitudinal velocity component away from the system intake:

$$x \to \infty, \; v_1(x, 0) \to 2^{-2/3} U l^{-1/3} b^{2/3} x^{-1/3} \tag{4.11}$$

which shows that for a chain of plates the damping of velocity is slower than the corresponding rate of concentration decrease (2.12), as $x \to \infty$.

Formula (4.11) must obviously be treated with some circumspection, since it cannot be proved by a direct test similar to that applied in the case of convective diffusion in a chain of plates.

The authors thank Iu. P. Gupalo and Iu. S. Riazantsev for helpful discussions.

REFERENCES

1. VOSKANIAN A.B., GOLOVIN A.M., and TOLMACHEV V.V., Convective diffusion at low Reynolds numbers in a system of periodically distributed plates. PMTF, No.5, 1966.

2. GUPALO Iu.P., POLIANIN A.D., RIAZANTSEV Iu.S., and SERGEEV Iu.A., On convective mass transfer in a system of periodically distributed spheres. PMTF, No.4, 1979.

3. GUPALO Iu.P., POLIANIN A.D., and RIAZANTSEV Iu.S., On mass transfer between particles lying along the stream axis at high Péclet numbers. Izv. Akad. Nauk SSSR, MZhG, No.2, 1977.

4. POLIANIN A.D., On diffusion interaction of solid particles at high Péclet numbers. PMM, Vol.42, No.2, 1978.

5. FLYTZANI—STEPHANOPOULOS M., and SCHMIDT L.D., Evaporation rates and surface profiles on heterogeneous surfaces with mass transfer and surface reactions. Chem. Engng. Sci., Vol.34, No.3, p.365, 1979.

6. SCHLICHTING H., Theory of the Boundary Layer. Moscow, NAUKA, 1974.

7. SUTTON W.G.L., On the equation of diffusion in a turbulent medium. Proc. Roy. Soc., A., Vol.182, No.988, 1943.

8. BRAKHAGE H., NICKEL K., and RIEDER P. Auflösung der Abelschen Integralgleichung, 2 parts. Z. Angew. Math. u. Phys., Vol.16, No.2, p.295, 1965.

9. WHITTAKER E.T., and WATSON J. I., A Course of Modern Analysis, Pt.2 Cambridge Univ. Press, 1940.

10. OLMSTEAD W.E., and HANDELSMAN R.A., Asymptotic solution to a class of nonlinear Volterra integral equations, II. SIAM. J. Appl. Math., Vol.30, No.1, p.180, 1976.

Translated by J.J.D.